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Technical Note

Unsteady forced convection heat transfer on a flat plate embedded in the fluid-saturated porous medium with inertia effect and thermal dispersion

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Abstract

For an unsteady forced convection on a flat plate embedded in the fluid-saturated porous medium with inertia effect and thermal dispersion, this paper presents a precise and rigorous method to obtain the entire solutions from one-dimensional transient conduction ($\xi = 0$) to steady forced convection in porous medium ($\xi = 1$) under conditions of uniform wall temperature and uniform heat flux, respectively. It is worth noted that the rate of unsteady heat transfer can be accelerated by the thermal dispersion, which may be regarded as the effect of mixing or agitating, to enhance the heat transfer in porous medium. Additionally, it is found that the time response, from the transient heat conduction to a steady forced convection in Darcy's flow, is $\tau = 1$, and is independent of wall heating condition and thermal dispersion strength (ϕ).

1. Introduction

For such wide applications, as geothermal survey and designs of high temperature insulation, packed reactor or absorbent, and thin film separation in chemical processes, a large number of studies have been conducted on the heat and mass transfer in porous medium [1].

Owing to the adding time effect, the transient heat transfer is usually difficult to solve with either an analytical approach or numerical method. Therefore, most results were limited in the area of steady thermal convection, and only a few solutions were obtained for the case of non-steady state in the previous work. As for unsteady forced convection in a porous medium, by means of a second-order upwind, Kimura [2] solved the problem of transient heat transfer in Darcy flow. Nakayma and Ebinuma [3] studied the inertia effect on the transient forced convection for a suddenly heated plate by using the Forchheimer-extended Darcy law, where the fluid flowing starts at the same time when the heating starts, and applied a quasi-similarity transformation. Additionally, with a scale analysis, Bejan and Nield [4] obtained solutions for three regimes, namely, the initial stage, transient period, and steady state during unsteady heat transfer in Darcy flow.

The aim of the present work is to provide a precise and rigorous method to study the transient forced convection heat transfer in the Forchheimer-extended Darcy flow that is steady, parallel and uniform as the embedded heated plate is suddenly changed with a uniform wall temperature or uniform heat flux, and to quantify the effect of inertia force on the intermediate regime of unsteady forced convection in a porous medium. Moreover, the object of this study is to refine the results presented in [4] in order to take into account the effect of thermal dispersion on the transient heat transfer in Darcy flow. According to author's knowledge, this case has not been investigated yet.

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Nomenclature

C	constant in Eq. (5)
C_p	specific heat of fluids in Eqs. (3) and (4) (J/(m K))
C_S	specific heat of porous medium in Eq. (3) (J/(m K))
D	particle size in Eq. (5) (m)
f	dimensionless velocity, $=u/u_D$
h	local heat transfer coefficient (J/(s m^2 K))
k	thermal conductivity of fluid (J/(s m K))
k'	thermal dispersion conductivity of porous medium (J/(s m K))
k_e	effective thermal conductivity in Eq. (7.2), $=k+k'$ (J/(s m K))
k_s	thermal conductivity of porous medium (J/(s m K))
K, L	experiment parameter in Eq. (1)
Nu	local Nusselt number, $=hx/k$
P	dynamic pressure of fluids (kg$_f$/m^2)
Pe	local Peclet number, $=u_D x/\alpha$
Pe_D	local Peclet number based on particle size, $=u_D D/\alpha$
q_w	wall heat flux (J/(s m))
Re_L	inertial strength, $=u_D KL/v$
t	time (s)
T	temperature (K)
T_w	wall temperature (K)
T_∞	ambient temperature (K)
u	velocity component in the x-direction (m/s)
u_D	average velocity for Darcy flow (m/s)
x	coordinate along the wall surface (m)
y	coordinate normal to the wall surface (m)

Greek symbols

α	effective molecular diffusivity defined in Eq. (4) (m^2/s)
α'	thermal dispersion diffusivity along y coordinate defined in Eq. (5) (m^2/s)
α_e	effective thermal diffusivity, $=\alpha+\alpha'$ (m^2/s)
γ	normalized inertia strength defined in Eq. (21)
δ	the thickness scale of dynamic thermal layer proposed in Eq. (10) (m)
δ_s	a steady thermal layer, $\sim x/Pe^{1/2}$ (m)
δ_t	the initial thermal layer, $\sim(\alpha t/\sigma)^{1/2}$ (m)
ε	porosity in Eqs. (3) and (4)
ζ	pseudo-similarity coordinate defined in Eq. (12)
η	similarity coordinate defined in Eq. (22)
θ	dimensionless temperature defined in Eqs. (16.1) and (16.2)
λ	combined variable defined in Eq. (13)
μ	viscosity of fluids in Eq. (1) (kg/(m s))
ξ	dimensionless time defined in Eq. (11)
ρ	density of fluids in Eqs. (3) and (4) (kg/m^3)
ρ_s	density of porous medium in Eq. (3) (kg/m^3)
σ	thermal capacity ratio defined in Eq. (3)
τ	traditional dimensionless time, $=u_D t/(x\sigma)$
φ	index in Eqs. (14) and (15)
Φ	dispersion strength, $=CPe_D$

Subscripts

s	steady forced convection
t	transient heat conduction
∞	a far from wall surface

2. Formulation of problem

Consider a laminar flow of incompressible Newtonian fluid with constant properties over a semi-infinite plate in a porous medium. The plate is suddenly heated and, subsequently, maintains either uniform temperature (UWT), T_w, or constant heat flux (UHF), q_w, over the surface. Moreover, the system is assumed that:

1. the porous medium is isotropic, homogeneous, and saturated with fluid;
2. a local thermal equilibrium in system can be realized;
3. momentum boundary layer is negligible;
4. thermal boundary layer is thin;
5. the conjugate effect on a plate is not considered.

As a result, the transient heat transfer in Forchheimer-extended Darcy flow that is steady, parallel and uniform may be mathematically described by

1. *Momentum equation*

$$\frac{dp}{dx}=\frac{\mu}{K}u+L\rho u^2 \tag{1}$$

in which the pressure drop, $-dp/dx$, can be induced from Darcy flow and be expressed as $\mu u_D/K$ [1] where u_D is termed as an average velocity in Darcy flow [5]. Additionally, both parameters K and L are experimental constants. It is noted that the velocity field is linearly related to the pressure drop when the inertia effect is not significant, that is, $L=0$.

2. *Energy equation*

$$\sigma\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}=\frac{\partial}{\partial y}\left(\alpha_e\frac{\partial T}{\partial y}\right), \tag{2}$$

where thermal capacity ratio σ was defined as

$$\sigma = \frac{\varepsilon\rho C_p + (1-\varepsilon)\rho_s C_s}{\rho C_p} \quad [1], \tag{3}$$

and $\alpha_e = (\alpha + \alpha')$ is an effective thermal diffusivity in which α is an effective molecular diffusivity termed as

$$\alpha = \frac{\varepsilon k + (1-\varepsilon)k_s}{\rho C_p}, \tag{4}$$

and α' that indicates thermal dispersion diffusivity along y coordinate was proposed by Plumb [6] as

$$\alpha' = \frac{k'}{\rho C_p} = CuD, \tag{5}$$

where C is a constant ranging from 1/3 to 1/7.

The initial and boundary conditions to satisfy Eq. (2) can be given as

$$T(x, y, t \leqslant 0) = T_\infty, \tag{6}$$

$$T(x, 0, t > 0) = T_w \quad \text{(UWT)} \tag{7.1}$$

or

$$-k_e\left(\frac{\partial T}{\partial y}\right)_{(x,0,t>0)} = q_w \quad \text{(UHF)}, \tag{7.2}$$

and

$$T(x, \infty, t) = T_\infty. \tag{8}$$

In condition (7.2), $k_e = k + k'$ is an effective thermal conductivity [7]. Note that Eq. (7.2) will be reduced as $k_e = k$ when the thermal dispersion effect is neglected to investigate the inertia effect on transient heat transfer over a horizontal surface in Forchheimer-extended Darcy flow.

Define a dimensionless velocity, $f = u/u_D$, then Eq. (1) can be recast into

$$Re_L f^2 + f - 1 = 0, \tag{9}$$

where $Re_L = u_D KL/\nu$ expresses the inertia strength in Forchheimer-extended Darcy flow. Note that Eq. (9) may be reduced to $f = 1$ if $Re_L = 0$ for a Darcy flow model.

3. Scale analysis of dynamic thermal layer and transformed variables

From the scale analysis of Eq. (2), unsteady forced convection heat transfer is indeed dominated by one-dimensional transient heat conduction at initial state as $t \to 0$ and a steady forced convection at the final state as $t \to \infty$. Therefore, with the theory of thermal resistance in series, a thickness of dynamic thermal boundary layer can be proposed as

$$\delta(x,t) \sim \left(\frac{1}{\delta_t} + \frac{1}{\delta_s}\right)^{-1}, \tag{10}$$

where $1/\delta_t \sim 1/\sqrt{\alpha t/\sigma}$ and $1/\delta_s \sim 1/x/Pe^{1/2}$ indicate the thermal resistance at the initial ($t \to 0$) and final stages ($t \to \infty$), respectively.

To take Eq. (10) as a characteristic length, the dimensionless time and coordinate can be, respectively, defined as

1. *Dimensionless time*

$$\xi = \frac{\delta}{\delta_s} = \left(1 + \frac{x\big/\sqrt{\alpha t/\sigma}}{Pe^{1/2}}\right)^{-1}. \tag{11}$$

It is noted that the infinite time domain of $0 \leqslant t < \infty$ can be transferred into a finite zone of $0 \leqslant \xi \leqslant 1$ by using Eq. (11).

2. *Dimensionless coordinate*

$$\zeta = \frac{y}{\delta} = \frac{y}{x}\lambda, \tag{12}$$

where λ is a combined variable and defined as

$$\lambda = Pe^{1/2} + x/\sqrt{\alpha t/\sigma}. \tag{13}$$

4. Non-dimensional unsteady heat transfer equation

By an employment of Eqs. (11) and (12) and the definition of dimensionless velocity, Eq. (2) can be, respectively, recast, into

$$\theta'' + \frac{1}{2}\zeta\left[\xi^3 f + (1-\xi)^3\right]\theta' - \frac{\varphi}{2}\left[(1-\xi)^3 + \xi^3 f\right]\theta = \frac{1}{2}\xi(1-\xi)\left[(1-\xi)^2 - \xi^2 f\right]\frac{\partial\theta}{\partial\xi} \tag{14}$$

and

$$(1 + CPe_d)\theta'' + \frac{1}{2}\zeta\left[\xi^3 + (1-\xi)^3\right]\theta' - \frac{\varphi}{2}\left[(1-\xi)^3 + \xi^3\right]\theta = \frac{1}{2}\xi(1-\xi)\left[(1-\xi)^2 - \xi^2\right]\frac{\partial\theta}{\partial\xi} \tag{15}$$

for effects of inertia force and thermal dispersion on an unsteady forced convection heat transfer in a porous medium. In Eqs. (14) and (15), dimensionless temperature θ is defined as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \quad \text{(UWT)}, \tag{16.1}$$

when index $\varphi = 0$; and

$$\theta = \frac{T - T_\infty}{q_w x/k}\lambda \quad \text{(UHF)} \tag{16.2}$$

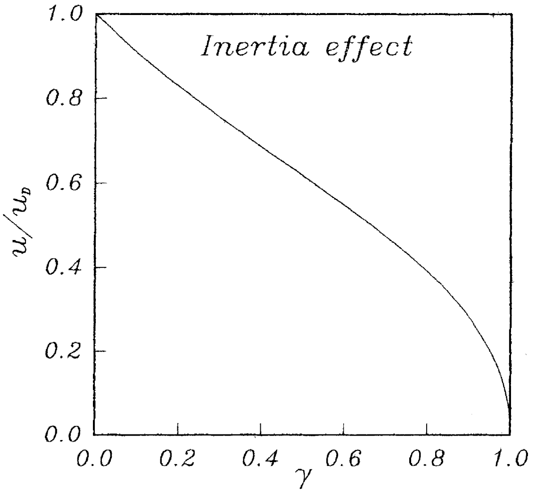

Fig. 1. The dimensionless velocity varying with normalized inertia strength γ in Forchheimer-extended Darcy flow.

when index $\varphi = 1$, respectively. Additionally, a notation of "′" expresses the differentiation with respect to variable ζ.

The initial and boundary conditions for Eqs. (14) and (15) can be rewritten as

$$\theta(\xi, 0) = 0, \tag{17}$$

$$\theta(\xi, 0) = 1 \quad \text{(UWT)} \quad \text{or} \quad \theta'(\xi, 0) = -1 \quad \text{(UHF)}, \tag{18}$$

$$\theta(\xi, \infty) = 0. \tag{19}$$

5. Numerical method

Non-similarity equations (14) and (15) associated with conditions (17)–(19) and velocities depicted in

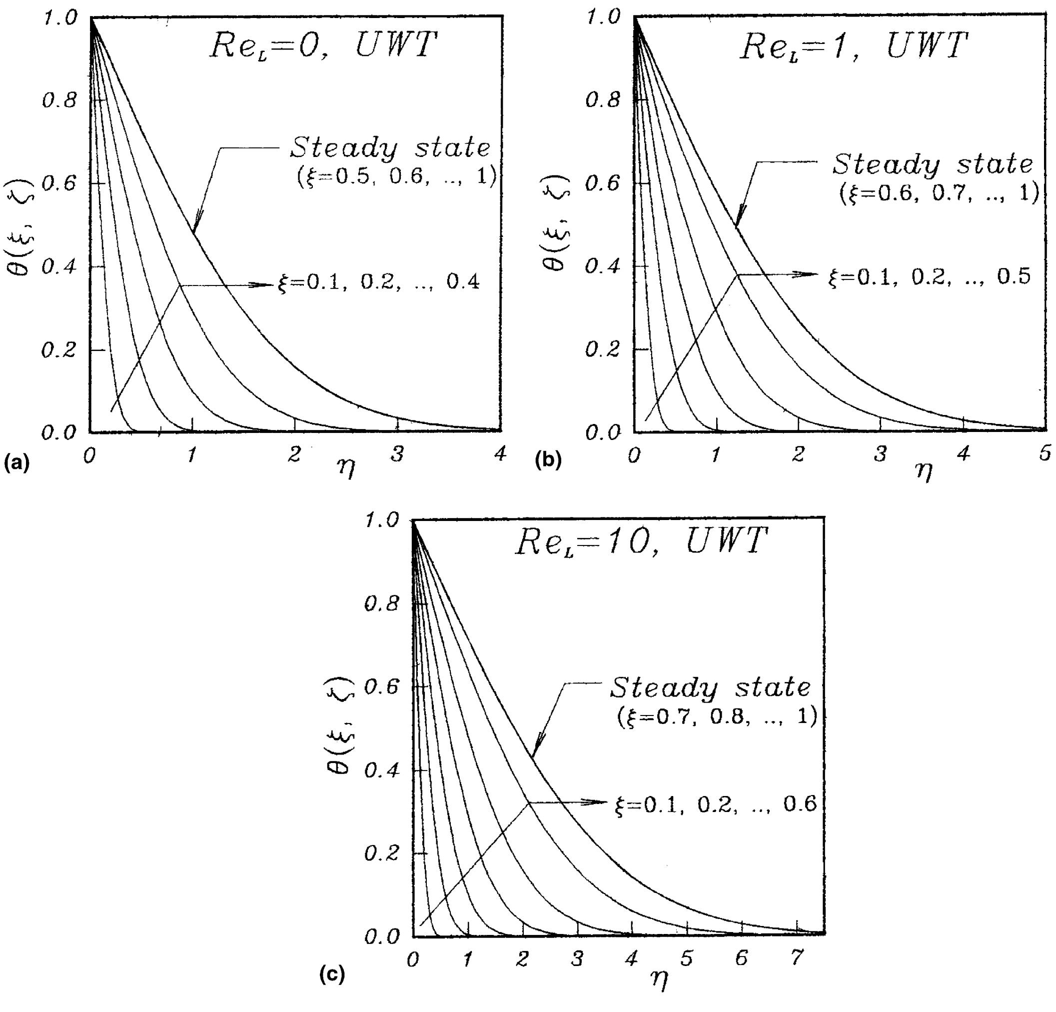

Fig. 2. Thermal boundary layer growth in a porous medium with inertia effect under condition of UWT.

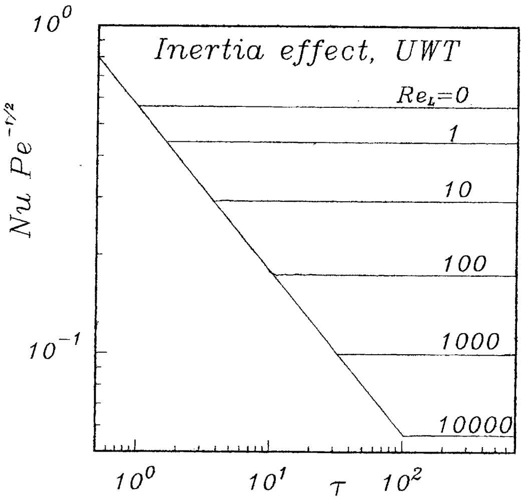

Fig. 3. The rate of unsteady heat transfer in a porous medium with inertia effect under condition of UWT.

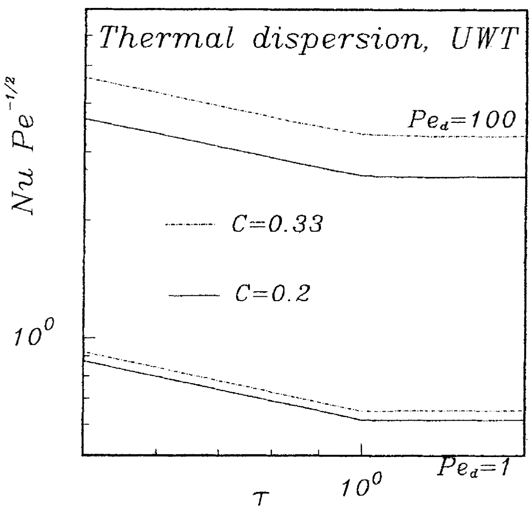

Fig. 5. The rate of unsteady heat transfer in a porous medium with thermal dispersion under condition of UWT.

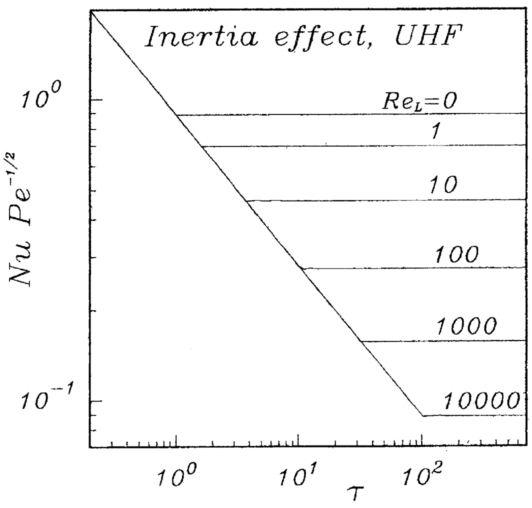

Fig. 4. The rate of unsteady heat transfer in a porous medium with inertia effect under condition of UHF.

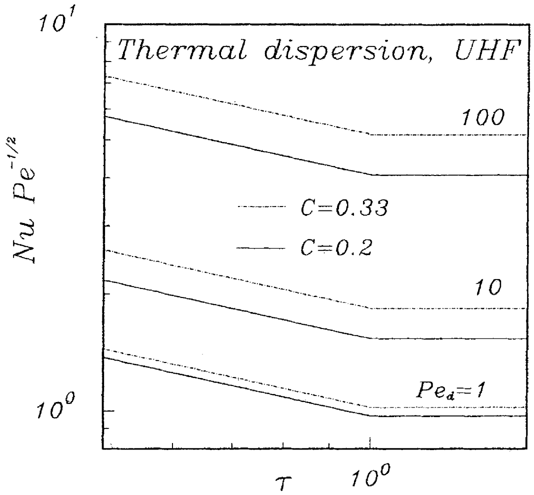

Fig. 6. The rate of unsteady heat transfer in a porous medium with inertia effect under condition of UHF.

Eq. (9) are both solved from the case of transient heat conduction ($\xi = 0$) to another case of a steady forced convection ($\xi = 1$) under conditions of UWT and UHF, respectively, by a method of lines (MOLs) combined with the central finite difference and Newton's iterations. This fully implicit scheme is special for the coefficient of $\partial\theta/\partial\xi$, in which the value can be changed from positive to negative with increasing dimensionless time, in the right-hand side of Eqs. (14) and (15). The details of the calculation procedure have been documented by Cheng [8].

6. Results and discussion

6.1. With inertia effect

By solving Eq. (9), the inertia effect on velocities in a porous medium may be described by an exact solution as

$$f = \frac{-1 + \sqrt{1 + 4Re_L}}{2Re_L}, \tag{20}$$

where Re_L is termed as the inertia strength. To clarify a variation of the velocity field in Forchheimer-extended Darcy flow, the inertia strength is normalized by

$$\gamma = \left(1 + \frac{1}{Re_L}\right)^{-1} \quad (21)$$

so that the range of $0 \leqslant Re_L < \infty$ can be transferred into the finite area of $0 \leqslant \gamma \leqslant 1$. As plotted in Fig. 1, it is obvious that the dimensionless velocity decreases from one to zero in a porous medium with inertia effect. This is in agreement with a conclusion in [3].

By a view of Eq. (14), the inertia effect on the unsteady forced convection is governed by parameter of Re_L. For cases of $Re_L = 0$, 1, and 10, Fig. 2 displays that dimensionless temperature θ is together with the similarity coordinate

$$\eta = \frac{y}{x} Pe^{1/2} = \zeta\xi \quad (22)$$

under condition of UWT. As exhibited in this figure, the thermal layer gradually grows with time, and the thickness of the thermal layer arriving in a steady state increases with values of Re_L from 4 to 10. In other words, the rate of transient heat transfer will be reduced if the pore size of medium is made largely, or packing material is arranged at random in the tower. Figs. 3 and 4 depict the values of

$$\frac{Nu}{Pe^{1/2}} = \frac{-\theta'(\xi, 0)}{\xi} \quad \text{(UWT)} \quad (23)$$

and

$$\frac{Nu}{Pe^{1/2}} = \frac{1}{\xi\theta(\xi, 0)} \quad \text{(UHF)}, \quad (24)$$

respectively, varying with time $\tau = u_D t/x$. It is clearly seen that time response from one-dimensional transient heat conduction to a steady forced convection in a porous medium is increased with the inertia strength.

6.2. With thermal dispersion

Transient heat transfer in Darcy flow with thermal dispersion is dominated by values of CPe_D, called the

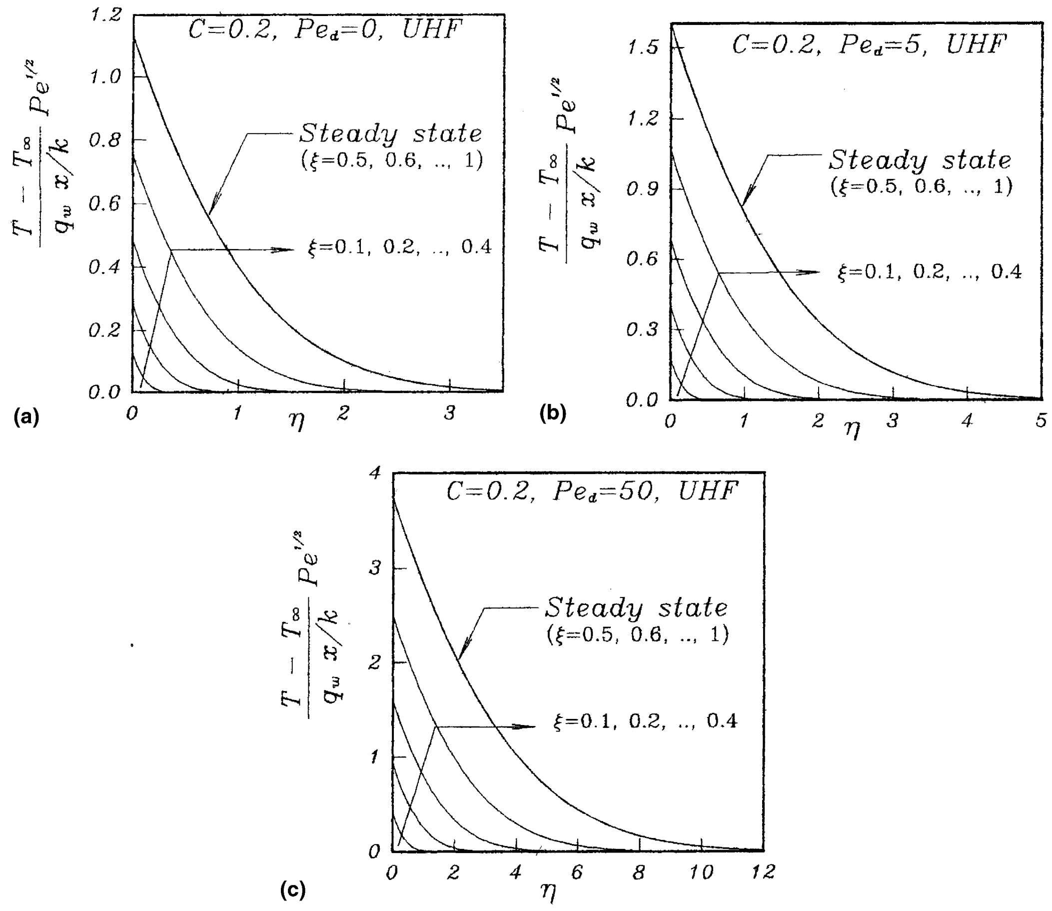

Fig. 7. Thermal boundary layer growth in a porous medium with thermal dispersion under condition of UHF.

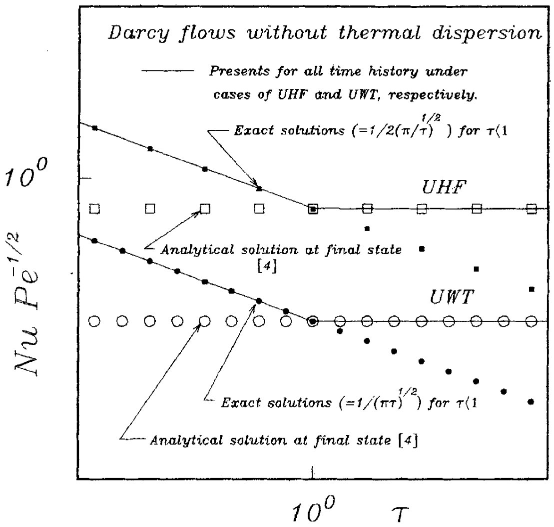

Fig. 8. A comparison of numerical solutions and the exact solutions at the initial and final stages for transient forced convection in Darcy flow without thermal dispersion under conditions of UWT and UHF.

thermal dispersion strength and remarked as ϕ, as shown in Figs. 5 and 6 under conditions of UWT and UHF, respectively. Obviously, these diagrams illustrate that the rate of unsteady heat transfer can be accelerated by the thermal dispersion that may be regarded as the effect of mixing or agitating to enhance the heat transfer in a porous medium. Additionally, as known from these two figures, it is found that the time required for the system to change from the transient heat conduction to a steady forced convection in Darcy flow is $\tau = 1$ and independent of wall heating condition and thermal dispersion. Fig. 7 shows dimensionless temperature θ vs. the coordinate η under condition of UHF. As demonstrated in cases of $\Phi = 0$, 1, and 10, the time for thermal boundary layer at a steady state are both $\xi = 0.5$. This trend is consistent with the result in Fig. 6.

For the case of Darcy flow without thermal dispersion under conditions of UWT and UHF, as analyzed by Bejan and Nield [4], the full numerical solutions are also presented to compare with the exact solutions at the initial and final stages, respectively. As shown in Fig. 8, the present solutions are very close to the analytical solutions at the limiting stages of time-dependent heat conduction and a steady forced convection in a porous medium.

7. Conclusion

A rigorous and precise method has been successful in obtaining complete and accurate solutions from the transient heat conduction at $\xi = 0$ to a steady forced convection in a porous medium at $\xi = 1$ so that the heat rate in the middle of unsteady forced convection in a porous medium with the inertia effect and thermal dispersion can be, respectively, examined in detail. It is worth noting that the effect of thermal dispersion on transient heat transfer in Darcy flow is first presented to be investigated in this work.

Acknowledgements

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